

# Final Exam – Review 3 – Problems

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## 1 Definition of volume

### Problem 1

The base of a solid is a square with vertices at  $(1, 0)$ ,  $(0, -1)$ ,  $(-1, 0)$ ,  $(0, 1)$ . Each cross-section perpendicular to the  $x$ -axis is a semicircle. Find the volume of the solid.

## 2 Disk method

### Problem 2

The region in the first quadrant enclosed by the line  $y = 2a - ax$  (where  $a > 0$ ) is rotated about the  $x$ -axis. Find the volume of the resulting solid.

## 3 Prof. Steel's favorite problem

### Problem 3

The surface of a bowl is obtained by rotating the curve  $x = \ln(y + 4)$  for  $y \geq 0$  about the  $y$ -axis. Water pours into the bowl at a constant rate of 3 cubic units/min. How fast is the water level rising when the water is  $e^2 - 4$  units deep?

## 4 Washer method

### Problem 4

Find a formula for the volume of the solid obtained by rotating the region bounded by  $y = x^3$  and  $y = x^2$ :

- (a) About  $y = 2$
- (b) About  $y = -1$
- (c) About  $x = -1$
- (d) About  $x = 2$

## 5 Shell method

### Problem 5

The region bounded by  $y = 2x^2$  and  $y = 3x - x^2$  is rotated about the line  $x = 2$ . What is the volume of the resulting solid?

### Problem 6

Find the volume of the ring obtained by drilling a cylindrical hole of radius  $r$  through the center of a ball of radius  $R$  (where  $r < R$ )

### Problem 7

(if time permits) Calculate the volume of the donut obtained by rotating the circle of radius 2 centered at  $(3, 0)$  about the  $y$ -axis